Gradiometric Analysis on a 3D Variable, Synthetic Density Model in Homogeneous and Inhomogeneous Medium

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1 Introduction

During an internship at the Trondheim based StatoilHydro Research Centre, a variable synthetic density model of a salt dome structure has been developed for forward modelling (fig. 1, table A).
With respect to the anomaly of its gravity potenital field, the following fields have been calculated:

- 3D cartesian components of **gravity**
- **Eötvös tensor** (full tensor gravity gradient, FTG)
- three **Invariants** (after Pedersen 1990)
- additional combined gradient fields

All fields were tested regarding the question to what extend such calculations could be suitable in order to gain or improve information about the structure of the subsurface.

2 Potential, Gravity, Gradients, Invariants

Referring to Newton's law of gravity which describes the force

$$F = G \cdot \frac{m_1 \cdot m_2}{r^2} \qquad (1)$$

between two masses m_1 and m_2 at distance r, where G=6,62428 m^3/(kg s^2) is the gravitational constant, and looking at a mass element

$$dm = \rho \cdot dV \qquad (2)$$

with ρ being its density with volume

$$dV = dx \cdot dy \cdot dz, \qquad (3)$$

the gravity potential is defined as

$$U = -G \cdot \int_x \int_y \int_z \rho \frac{dV}{r} \qquad (4)$$

From *(1)* and *(2)* follows directly that gravity variations depend on density. The model represents density variations in the subsurface and hence results in an anomaly of the gravitational field (fig.1).

The spatial derivatives of the 1st and 2nd order of the potential field are the gravity field

$$U_i = \frac{\partial U}{\partial i} \qquad (5)$$

and the gravity gradient tensor

$$U_{ij} = \frac{\partial^2 U}{\partial i \partial j} \qquad (6)$$

where i,j=x,y,z .

The symmetric gravity gradient tensor (Eötvös Tensor) consists of 3x3 elements, five of them independent. Pedersen (1990) showed three spatial invariants which are derived from the tensor elements:

$$I_0 = \nabla^2 U = trace(H(U)) = U_{xx} + U_{yy} + U_{zz} = 0 \quad (Laplace) \qquad (7)$$

$$I_1 = U_{xx}U_{yy} + U_{yy}U_{zz} + U_{xx}U_{zz} - U_{xy}^2 - U_{yz}^2 - U_{xz}^2 \qquad (8)$$

$$I_2 = Det(H(U))$$
$$= U_{xx}(U_{yy}U_{zz} - U_{yz}^2) + U_{xy}(U_{yz}U_{xz} - U_{xy}U_{zz}) + U_{xz}(U_{xy}U_{yz} - U_{xz}U_{yy}) \qquad (9)$$

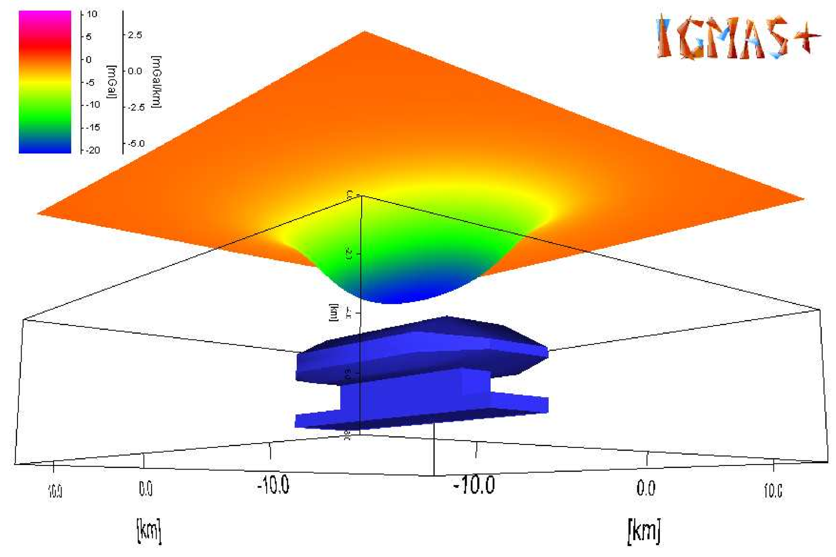

Fig.1 - Synthetic density model and gravitational anomaly (U_z)

The invariant fields are independent of the coordinate system and thus provide a useful visualization of gravity anomalies. A highpass filter like effect is caused by the multiplication when calculating I_1 and I_2. This will however converge the anomalies of the field further towards the projection of relevant density contrasts (comp. fig. 2).

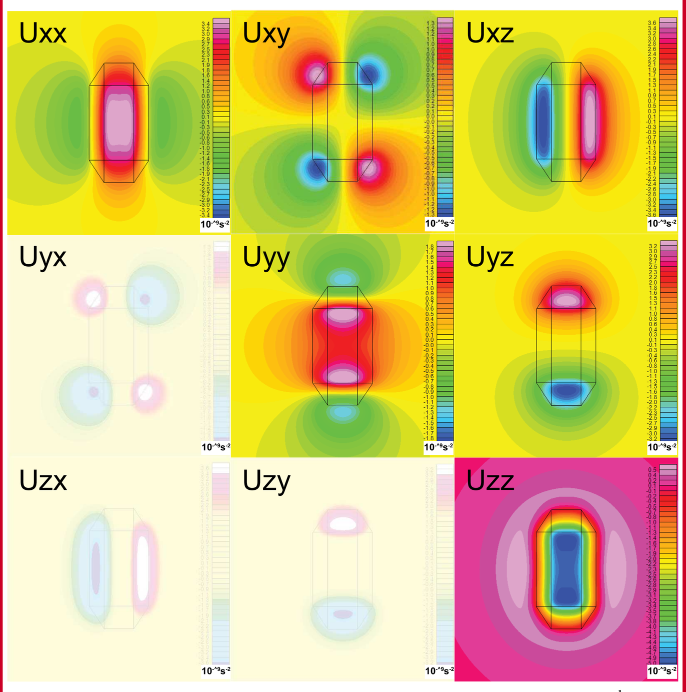

Fig. 5 – The gravity gradient tensor (Eötvös tensor) is the 2nd spatial derivative, or Hessian matrix, of the gravity potential. This figure refers to model option *a*.

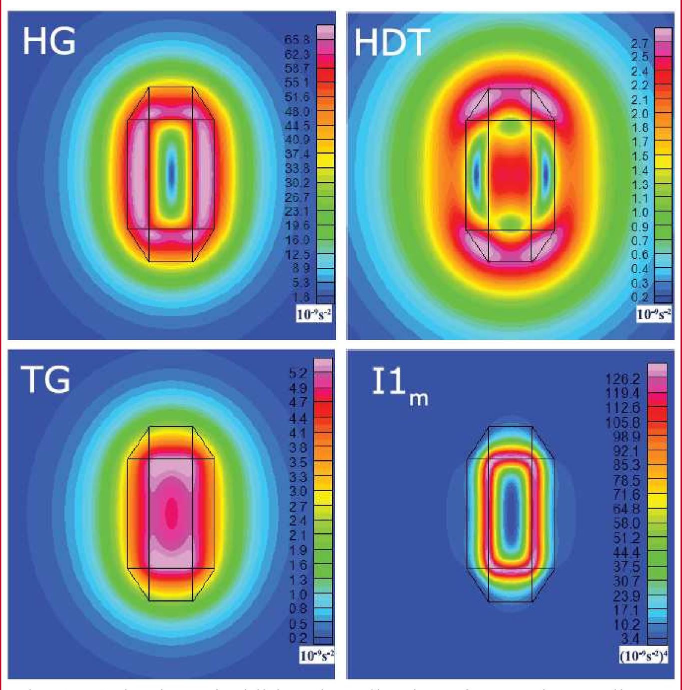

Fig. 6 – Selection of additional applications for gravity gradients. HG: horizontal gradient magnitude of Uz; HDT: horizontal directive tendency (differential curvature magnitude); TG: total gradient magnitude of Uz; I1m: experimental gradient combination (multiplicatively I1). All figures for model option *a*.

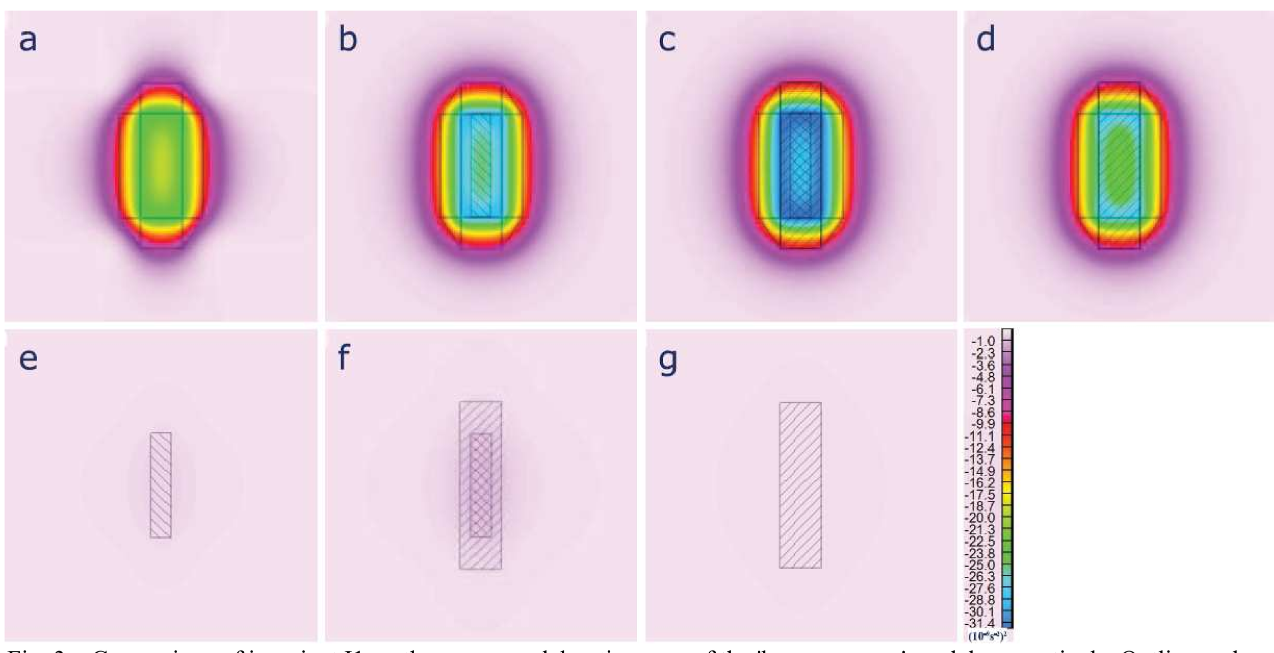

Fig. 3 – Comparison of invariant I1 on the seven model options a-g of the 'homogeneous' model, respectively. Outline and extend of the low-density regions are marked by dark solid lines and hatched areas.

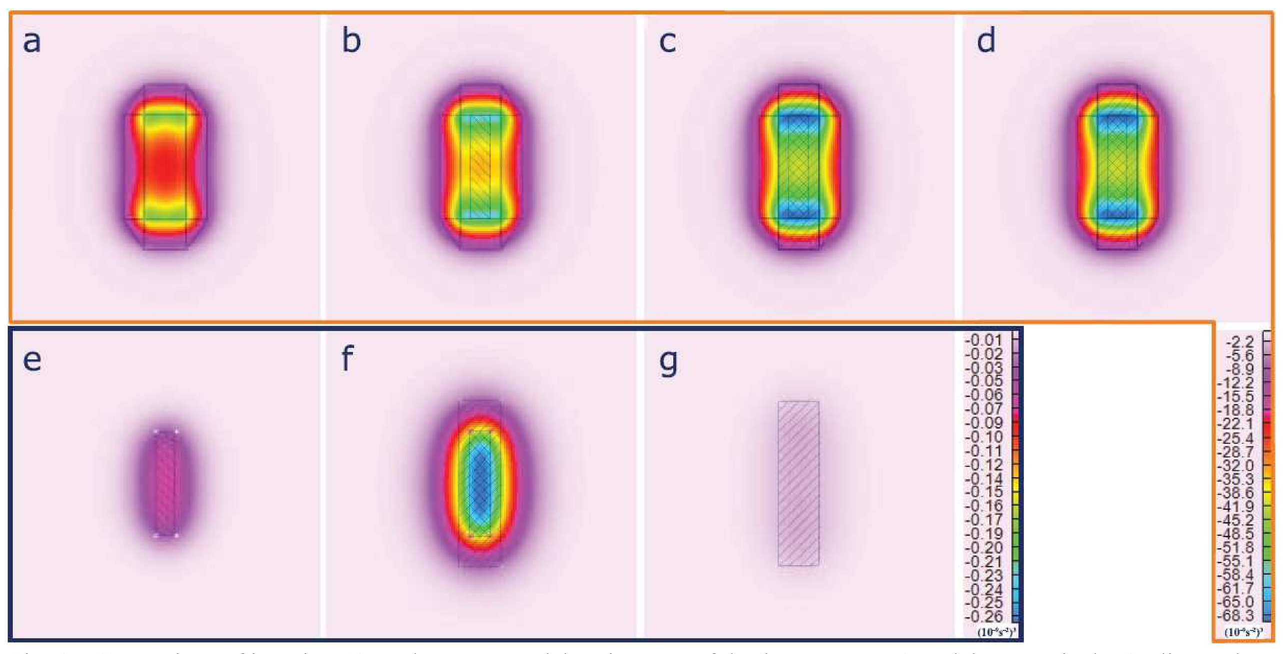

Fig. 4 – Comparison of invariant I2 on the seven model options a-g of the 'homogeneous' model, respectively. Outline and extend of the low-density regions are marked by dark solid lines and hatched areas. Please notice the two different colour scales due to the extreme highpass-filter-like effect when multiplicatively calculating I2 from the gradients.

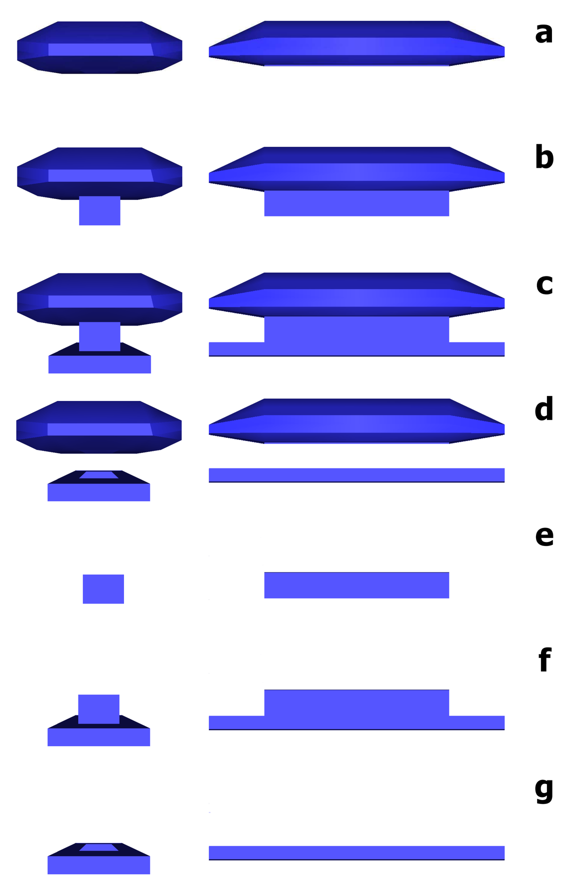

Table A – Seven possible model options a-g

3 The 'homogeneous' Model

The synthetic salt dome model is a well simplified model that satisfies the need for simple and easy-to-understand geometry rather than geological authenticity. It consists of nine vertical sections which are combined into a 3D model using triangulation. It was developed for calculating gravity with the Igmas modeling software (see Götze and Lahmeyer, 1988).

The 'homogeneous' model represents an embedded region of lower, sharp contrasted density in a homogeneous medium.

Table A shows all seven possible combinations (a,b,...,g) of the three parts which the 'salt dome' consists of. Varying this geometry leads to different calculated gravity anomaly signatures at the surface and hence results in different invariant fields, shown in figure 3 and 4. Figure 5 shows for model option *a* the full gravity gradient tensor (Eötvös tensor), from which all invariants can be calculated. Figure 6 shows four additional applications for combining gravity gradients – the horizontal gradient HG, differential curvature HDT, total gradient magnitude TG and an experimental gradient combination which acts again one multiplicative order higher than I2 and nearly perfectly traces the known extend of the subsurface body.

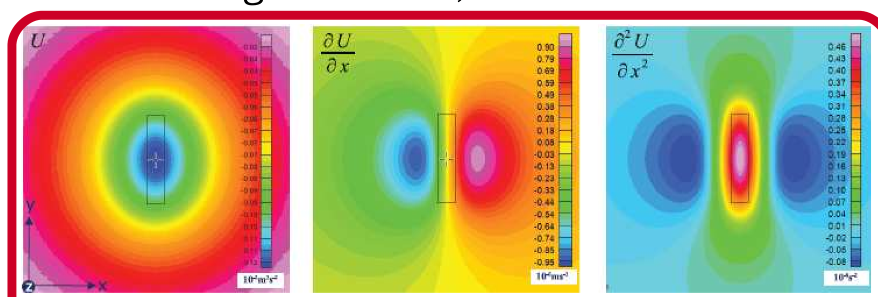

Fig. 2 – From left to right: shallow gravity potential field of the outlined body (low density); horizontal gravity in x; gravity gradient in xx. Notice the field lines converge towards the body outlines with increasing order of derivative. Simultaneously, the signal amplitude decreases by 10^{-3} per order.

4 The 'inhomogeneous' Model

In order to test the understanding of the tensor and its 'derivatives' in a more realistic scenario, the salt dome structure was deformed and set in a inhomogeneous, layered medium. Figure 7 shows that some features of the synthetic salt dome can clearly be found in the calculated field maps, e.g. easily slanted areas in the vertical gradient map in green (fig. 7, upper map). Simply looking at some gradient components must always be done with caution since the tensor elements are dependent on a coordinate system. Calculating invariant fields can significantly improve the process of using and understanding gravity gradient fields. Another interesting feature in the 'realistic' model is a change in polarity due to a density cross-over. This occurs when a body of relatively constant density (salt dome etc.) merges vertically from a region of relatively higher density into a layer of relatively lower density (see e.g. Saad, 1990).

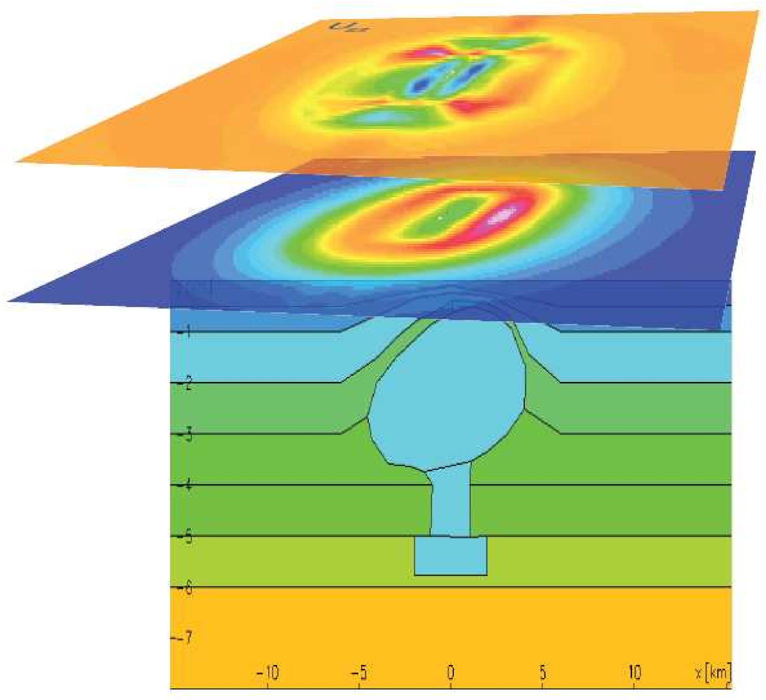

Fig. 7 – Advanced model in inhomogeneous, layered medium; calculated gravity gradient Uzz (upper map) and horizontal gradient (lower map).

5 Results

The results of this qualitative study support the fact that using gravity gradients – and particularly their invariants – allows for more accurate determination of borders and structure of density anomalies when compared to 'normal' gravity modelling. While invariant I1 rather emphasises the general shape of a density variation, invariant I2 seems to point out spots where the anomaly notably changes in all 3 spatial dimensions. Higher multiplicity in I2 results in a highpass-filter-like weakening of signals from deep below the surface. Interpreting gravity gradients with anomalies over a more realistic, inhomogeneous medium is unsurprisingly more sophisticated. However, density cross-over regions can easily be found.

6 Outlook

- spectral analysis
- applying wavelength filtering
- Deepen topic in a 3D-cave, using stereo projection system
- software implementation of invariants (IGMAS+)
- density cross-over detection
- what about magnetics?

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